Physica A 391 (2012) 3257–3265

Contents lists available at SciVerse ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Disorder induced phase transition in kinetic models of opinion dynamics

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ARTICLE INFO

Article history:
Received 2 September 2011
Received in revised form 12 December 2011
Available online 3 February 2012

Keywords:
Social system
Consensus
Mean field
Critical phenomenon

ABSTRACT

We propose a model of continuous opinion dynamics, where mutual interactions can be both positive and negative. Different types of distributions for the interactions, all characterized by a single parameter p denoting the fraction of negative interactions, are considered. Results from exact calculation of a discrete version and numerical simulations of the continuous version of the model indicate the existence of a universal continuous phase transition at $p = p_c$ below which a consensus is reached. Although the order–disorder transition is analogous to a ferromagnetic–paramagnetic phase transition with comparable critical exponents, the model is characterized by some distinctive features relevant to a social system.

1. Introduction

Quantitative understanding of individual and social dynamics has been explored on a large scale [1–8] in recent times. Social systems offer some of the richest complex dynamical systems, which can be studied using the standard tools of statistical physics. With the availability of data sets and records on the increase, microscopic models mimicking these systems help in understanding their underlying dynamics. On the other hand, some of these models exhibit novel critical behavior, enriching the theoretical aspect of these studies.

Mathematical formulations of such social behavior have helped us to understand how global consensus (i.e., agreement of opinions) emerges out of individual opinions [9–25]. Opinions are usually modeled as variables, discrete or continuous, and are subject to spontaneous changes as well as changes due to binary interactions, global feedback and even external factors. Apart from the dynamics, the interest in these studies also lies in the distinct steady state properties: a phase characterized by individuals with widely different opinions and another phase with a major fraction of individuals with similar opinions. Often the phase transitions are driven by appropriate parameters of the model.

In this paper we study a model of opinion dynamics by considering two-agent interactions. Continuous opinion dynamics has been studied for a long time [26–28], with the models designed in such a way that eventually the opinions cluster around one (consensus), two (polarization) or many (fragmentation) values. The average opinion or macroscopic behavior has been emphasized only in some recent works [23,24], where a phase transition from ordered to disordered phase has also been reported. However, in contrast to these models, we obtain here an ordered phase where even in the presence of a dominant opinion (symmetry broken phase), opposing opinions survive and a disordered phase where all opinion values coexist without any preference to any value (symmetric phase). Thus we present this in the general context of an order–disorder transition similar to that of the Ising and related models. We also compare our results with earlier works where a mean-field phase transition was observed in presence of contrarians in the society [21].

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doi:10.1016/j.physa.2012.01.046

The paper is organized as follows: in Section 2 we introduce the model. Then in Section 3 the main results are presented along with the calculations and numerical simulations. In Section 4 we extend the model to include bond dilution and present the phase diagram. Finally we discuss our results in Section 5.

2. The model

We propose a new model for emergence of consensus. Let $o_i(t)$ be the opinion of an individual i at time t. In a system of N individuals (referred to as the 'society' hereafter), opinions change out of pair-wise interactions:

$$o_i(t+1) = o_i(t) + \mu_{ij} o_j(t). \tag{1}$$

One considers a similar equation for $o_j(t+1)$. The choice of pairs $\{i, j\}$ is unrestricted, and hence our model is defined on a fully connected graph, or in other words, of infinite range. Note that this is simply a pair-wise interaction and we imply no sum over the index j. Here μ_{ij} are real, and it is like an interaction parameter representing the influence of the individual with whom interaction is taking place. The opinions are bounded, i.e., $-1 \le o_i(t) \le 1$. This bound, along with Eq. (1) defines the dynamics of the model. If, by following Eq. (1) the opinion value of an agent becomes higher (lower) than $+1$ (-1), then it is made equal to $+1$ (-1) to preserve this bound. The ordering in the system is measured by the quantity $O = \left|\sum_i o_i\right| / N$, the average opinion, which is the order parameter for the system.

The present model is similar in form to a class of simple models proposed recently [23–25,29], apparently inspired by the kinetic models of wealth exchange [30,31]. A spontaneous symmetry breaking was observed in such models: in the symmetry broken phase, the average opinion is nonzero while in the symmetric phase, the opinions of all individuals are identically zero indicating a 'neutral state'. The parameters representing conviction (self interaction) and influence (mutual interaction) in these models were considered either uniform (a scalar) or in the generalized case different for each individual, i.e, given by the components of a vector. In addition to this there is an added feature of the randomness in the influence term which in effect controls the sharpness of the phase transitions in these models.

In our proposed model, the conviction parameter or self interaction parameter is set equal to unity so that in absence of interactions, opinions remain frozen. In such a situation, it has been observed previously that any interaction, however small, leads to a highly unrealistic state of all individuals having extreme identical opinions (either $o_i = 1\ \forall i$ or $o_i = -1\ \forall i$) [24] when the interactions take up *positive values only*. This suggests that one should generalize the interactions to include both positive and negative values. This is realistic also in the sense that it reflects the fact that in a social interaction of two individuals, there may be either agreement or disagreement of opinions. We therefore consider not only a distribution of the values of μ_{ij} (to maintain the stochastic nature of the interactions) but also allow μ_{ij} to have negative values. We define a parameter p as the fraction of values of μ_{ij} which are negative, which, we will show later, leads to characteristic ordered and disordered states as in reality.

The fact that we allow random positive and negative values for the interactions may suggest that the model is analogous to a dynamic spin glass model [32,33], as in the latter, one can consider a dynamic equation for the spins which formally resembles Eq. (1). However, the two dynamic models are not equivalent with the following differences: (i) the interactions in the opinion dynamics models are never considered simultaneously and thus the question of competition leading to the possibility of frustration does not arise, and (ii) there is also no energy function to minimize, (iii) the symmetry $p \to 1-p$ does not exist in our model, which is naturally present for spin-glass. We will get back to the comparison of the two models in the context of phase transition later in this paper.

The effect of negative interactions was considered previously in a different opinion dynamics model under the name Galam contrarian [21]. The discrete, binary opinion model followed a deterministic evolution rule for a group of three or more individuals. It was shown that depending on the concentration of the contrarians, the system will either reach an ordered state, where there one of the opinions will have majority, or a disordered state, where no clear majority is observed. The critical behavior of the model is similar to the one we present here at least in the fully connected graph. However, our model considers continuous opinion values. Also, the Galam contrarians always take the opinion opposite to that of the majority. However, in our case we also consider the present state of opinion of the agents and accordingly even the discrete version of our model has three states. A two-state discrete version of this model will not show any ordered state.

3. Results

Unless otherwise mentioned, we keep μ_{ij} values within the interval $[-1, 1]$ for simplicity. In principle, several forms can be considered for μ_{ij} (annealed, quenched, symmetric, non-symmetric etc.). Further, there can be several distribution properties for μ_{ij} in the interval $[-1, 1]$ (discrete, piecewise uniform and continuous distributions). Unless otherwise stated, in our study, we would discuss the case when μ_{ij} are *annealed*, i.e., they change with time. In other words, at each pairwise interaction, the value of μ_{ij} is randomly chosen respecting the fact that it is negative with probability p. For this case, the issue of symmetry does not arise. We consider distributions for both continuous and discrete μ_{ij}.

In all the above cases, we find a symmetry breaking transition. Below a particular value p_c of the parameter p, the system orders (i.e., the order parameter O has a finite non-zero value), while the disordered phase (where $O = 0$) exists for higher values of p. Since this phase transition is very much like the thermally driven ferromagnetic–paramagnetic transition in magnetic systems, we have considered the scaling of the analogous static quantities, which are:

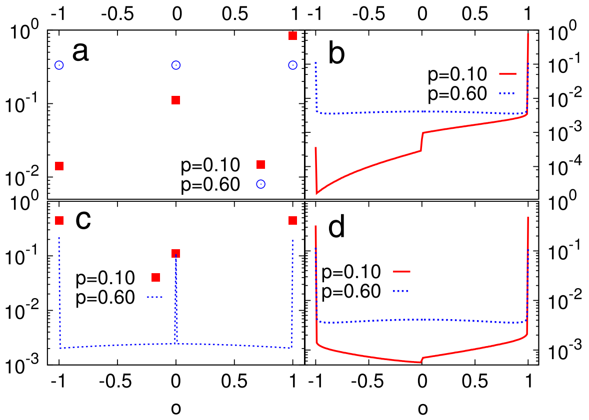

Fig. 1. Probability distribution of opinions for polarized (all +1) initial condition with (a) discrete μ_{ij} ($p_c = 1/4$) and (b) continuous μ_{ij} ($p_c \approx 0.34$); same for random (uniform between $[-1:+1]$) initial condition with (c) discrete μ_{ij}($p_c = 1/4$) and (d) continuous μ_{ij} ($p_c \approx 0.34$). All data are for $N = 4096$.

(i) the average order parameter $\langle O \rangle$, $\langle \cdots \rangle$ denoting average over configurations,
(ii) the fourth order Binder cumulant $U = 1 - \frac{\langle O^4 \rangle}{3\langle O^2 \rangle^2}$,
(iii) a quantity analogous to susceptibility per spin, which we write as $V = N[\langle O^2 \rangle - \langle O \rangle^2]$.

We also calculate (iv) the condensate fraction $f_c = f_1 + f_{-1}$, where f_1 and f_{-1} denote the fraction of population with opinion 1 and -1 respectively. f_c is exclusive to this class of opinion dynamics models and is expected to show scaling behavior near the critical point [23,24].

Before a more detailed characterization of this transition, we find it useful to describe the behavior of the probability distribution of the order parameter first. This distribution itself shows the signature of a phase transition. We consider both the cases with discrete (± 1) and continuous (within the bound $-1 \leq \mu_{ij} \leq +1$) values for μ_{ij}. For each of the above case we consider both polarized and random initial conditions. We show the cases where the initial condition is fully polarized (all $+1$) and μ_{ij} are discrete (Fig. 1(a)) and continuous (Fig. 1(b)). We consider two p values, one below and one above the critical point. As can be seen from the figures, for $p = 0.6 > p_c$ the distributions are symmetric in both the cases, which manifestly imply disorder, while the ordered phase is asymmetric with a bias towards $+1$ due to the initial condition.

Now consider the case when the initial condition is random instead of being polarized as above. Then of course in a particular ordered state either majority positive or majority negative opinion can occur. Hence the order parameter distributions (after averages over many configurations) become symmetric (Fig. 1(c) and (d)) even in the ordered phase for both discrete and continuous μ_{ij} values. An interesting point, however, is the depletion of population in the intermediate (between the extremist with opinions ± 1) opinion values in an ordered society (Fig. 1(c)). This is even more pronounced when we make μ_{ij} values discrete (± 1). In this case, the distribution becomes discretized almost simultaneously with the symmetry breaking transition such that the intermediate opinion values do not exist at all in the ordered state (Fig. 1(d)). Therefore, we find that in the ordered state with discrete μ_{ij}, the society becomes highly clustered in the sense that continuous variation of opinion is no longer a possibility. This of course is commensurate with the continuous μ_{ij} case described above, where possibility of ordering leaves the society highly clustered as is generally seen before and after decisive elections.

It is important to note here that although the distribution of average opinion is sensitive to the initial condition, the order parameter itself is not as the latter is obtained by taking the absolute value of the average opinion in a single configuration and then taking a further average over all configurations in a numerical study. Moreover, the critical behavior is also unaffected by the changes in initial condition.

Our major interest lies in identifying the critical behavior in these models. Assuming that there exists a steady state for these models (which are numerically observed), one can derive exact expressions for the steady state probabilities f_1, f_0 and f_{-1} in the annealed *discrete* case where we assume the initial condition to be such that the agents have $o_i(t = 0) \in \{-1, 0, +1\}$. The scaling behavior of the order parameter and f_c can be exactly obtained in this method. To do that, we consider the probabilities that an agent's opinion gets decreased (for initial opinion $+1$ or 0) or increased (for initial opinion -1 or 0) or remains constant when two agents interact. For example, let us consider the change for the agent A, interacting with a second agent B (one can consider updated values of both but it does not matter): when both have opinion $+1$ (probability of occurrence f_1^2), A's opinion decreases with a probability p, giving the joint occurrence probability of the event as pf_1^2. On the other hand, when B has opinion -1, the similar event has the probability $(1-p)f_1f_{-1}$. One can similarly find out the probabilities of decrease and increase in all possible cases. The exact expression for the net increase probability is $pf_{-1}^2 + (1-p)f_1f_0 + pf_0f_{-1} + (1-p)f_1f_{-1}$ and that for the net decrease probability is $pf_1^2 + pf_1f_0 + (1-p)f_0f_{-1} + (1-p)f_1f_{-1}$. In the steady state these two should be equal, i.e.,

$$pf_1^2 + pf_1f_0 + (1-p)f_0f_{-1} = pf_{-1}^2 + (1-p)f_1f_0 + pf_0f_{-1}, \quad (2)$$

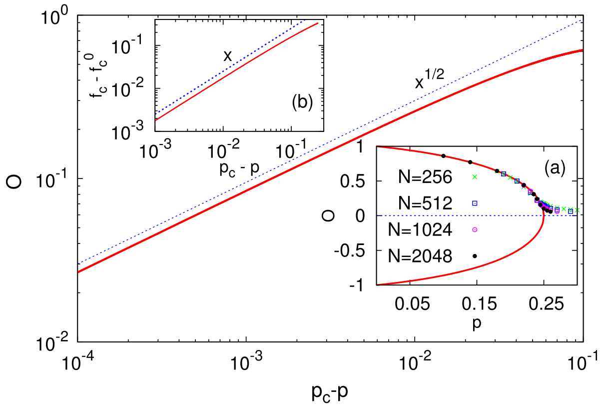

Fig. 2. Discrete μ_{ij}: power law behavior of the order parameter O near the critical point p_c (Eq. (8)) showing $\beta = 1/2$. The dotted line is $x^{1/2}$, a visual guide. Inset: (a) Phase diagram. The points represent simulation results. They are in better agreement with the analytical results as the system size is increased from 256 to 2048. (The lower half of the phase diagram follows from symmetry.) (b) Linear scaling of $f_c - f_c^0$. The dotted line is x^1.

which simplifies to

$$(2f_1 + f_0 - 1)\,[p - f_0(1-p)] = 0. \tag{3}$$

This means, either $2f_1 + f_0 = 1$, i.e., $f_1 = (1-f_0)/2 = f_{-1}$ which implies a disordered phase, or

$$f_0 = \frac{p}{1-p}. \tag{4}$$

Next we show that at criticality all three fractions would become equal such that $p_c = 1/4$. Let us take the solution in the disordered phase where $f_1 = f_{-1}$. We consider the processes contributing to the in/out flux for f_0. We enumerate all possibilities as before and get the following: flux into f_0 is $2[(1-f_0)/2]^2$ and flux out of f_0 is $f_0(1-f_0)$. So, in the steady state

$$\left(\frac{1-f_0}{2}\right)^2 = \frac{f_0(1-f_0)}{2}. \tag{5}$$

Hence, either $f_0 = 1$, which can be ignored by considering the steady states of the other two fractions, or $f_0 = 1/3$. Now, just at the critical point this solution will begin to be valid. Hence, at the critical point all three fractions are 1/3, leading to $p_c = 1/4$.

In the ordered phase, the order parameter O is given by $|f_1 - f_{-1}|$. To evaluate f_1 and f_{-1}, we calculate the flux in and out of f_1. Flux out of f_1 is $pf_1^2 + (1-p)f_1[1-(f_1+f_0)]$ and flux into f_1 is $(1-p)f_0f_1 + pf_0[1-(f_1+f_0)]$. Hence at steady state, f_1 is given by

$$f_1 = \frac{1 - 3p + 2p^2 \pm \sqrt{1 - 6p + 9p^2 - 4p^3}}{2(1 - 2p + p^2)}, \tag{6}$$

where we have used Eq. (4). Hence the order parameter is given by

$$O = \frac{1 - 3p + 2p^2 \pm \sqrt{1 - 6p + 9p^2 - 4p^3}}{(1-p)^2} + \frac{2p-1}{1-p}. \tag{7}$$

The variation of the order parameter with p is shown in Fig. 2. It shows the expected behavior, i.e., it vanishes at $p = p_c$. Rewriting the above equation in terms of $x = p_c - p$, algebraic simplifications give

$$O = \frac{3/8 + 2x + 2x^2 \pm \sqrt{9x/4 - 3x^2}}{9/16 + 3x/2 + x^2} - \frac{2x + 1/2}{3/4 + x}. \tag{8}$$

As $x \to 0$, $O \sim \sqrt{x}$, implying the critical exponent for O is $\beta = 1/2$. This also agrees well with the power law fit of the order parameter expression (Eq. (8)) near the critical point (Fig. 2). Calculation for f_c on the other hand shows that it has a constant value $f_c^0 = 2/3$ beyond p_c and for $p < p_c$, $f_c - f_c^0 \sim x^1$, i.e., vanishes linearly at p_c, which also perfectly agrees with numerical simulations. Numerical simulations for the model with continuous μ_{ij} yields similar behavior with $f_c^0 \simeq 0.22$ since opinion values other than ± 1 and 0 exist.

Using the same kind of argument as above, one can show that there will be no phase transition when $\mu_{ij} = \pm\mu_0$ with $\mu_0 \geq 2$. It is obvious that in this case the opinions can have only two values $+1$ and -1. Let f_1 be the fraction of agents

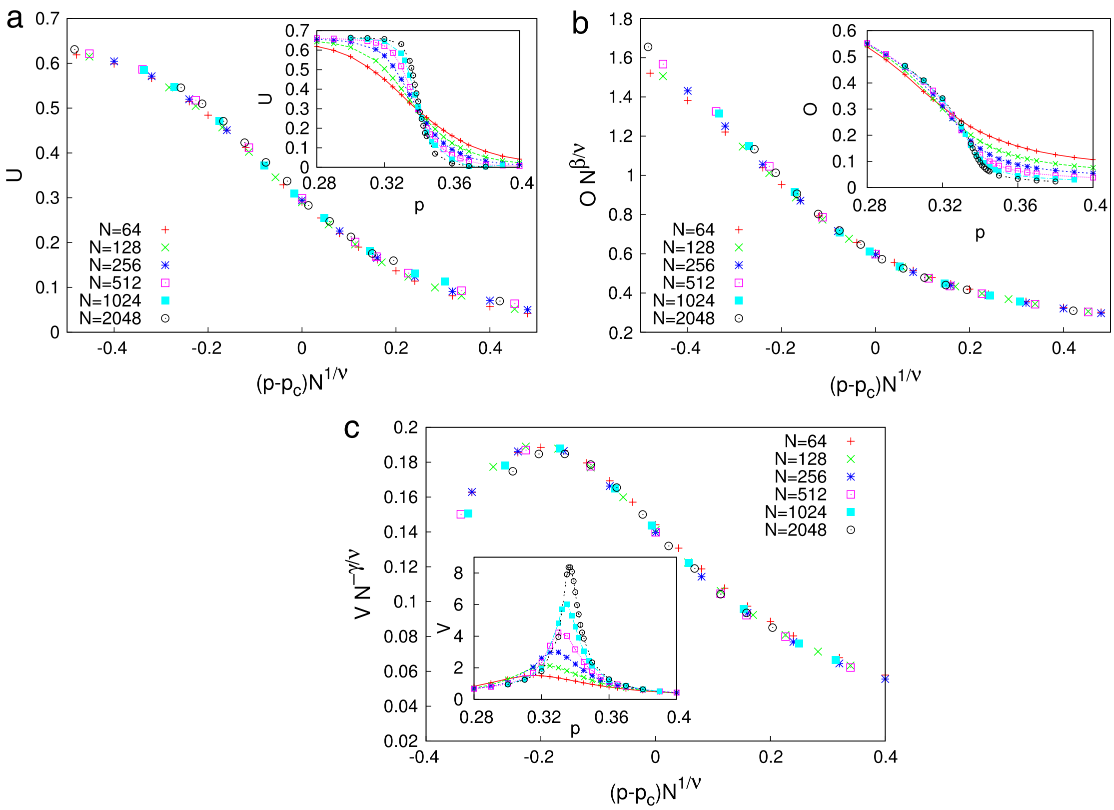

Fig. 3. Data for continuous, annealed μ_{ij} model, showing (a) finite size scaling of the Binder cumulant U for different system sizes N; the critical point is $p_c = 0.3404 \pm 0.0002$, and the best data collapse is for $\nu = 2.00 \pm 0.01$. Inset: Variation of U with p for different system sizes; (b) finite size scaling of order parameter O for different N; best data collapse is for $\beta = 0.50 \pm 0.01$. Inset: Variation of the order parameter O with p for different system sizes; (c) finite size scaling of V for different N; best data collapse is for $\gamma = 1.00 \pm 0.05$. Inset: Variation of V with p for different N. Number of averages for different system sizes are 3000 for $N = 256$, 1800 for $N = 512$, 1000 for $N = 1024$ and 400 for $N = 2048$.

having opinion +1. Once again considering net decreases and increases of opinions and equating them in the steady state, we get

$$p(1 - f_1)^2 = pf_1^2. \tag{9}$$

For any non-zero p the solution of this equation is $f_1 = 1/2$ thereby giving complete disorder. Hence, in this condition there cannot be an ordered phase for any finite p.

We performed Monte Carlo simulation for different system sizes ($N = 64, 256, 512, 1024, 2048$) to estimate p_c and all the relevant exponents for discrete as well as continuous μ_{ij}'s (see Fig. 3 where the data for the continuous distribution of μ_{ij} are presented). A Monte Carlo step is the simultaneous update of N agent's opinion values. For each simulation point, sufficient relaxation time was given (depending on system size), such that the measurable quantity reached a steady-state value. Then the ensemble average of those steady state values were taken (number of ensemble again depends on system size, see figure captions for details).

We estimated the critical point p_c from the crossing of the Binder cumulants for different system sizes [34]. Our estimate is $p_c \simeq 0.249 \pm 0.001$ for the discrete case which is consistent with the analytical value of 1/4 derived earlier. The critical Binder cumulant is $U^* = 0.30 \pm 0.01$. For the continuous case, $p_c = 0.3404 \pm 0.0002$, and $U^* = 0.284 \pm 0.004$. We find excellent finite size collapse for all cases. We estimate the correlation length exponent $\nu = 2.00 \pm 0.01$, the order parameter exponent $\beta = 0.50 \pm 0.01$ and the fluctuation exponent $\gamma = 1.00 \pm 0.05$.

One can consider several other distributions with a similar parameter p to test the universality of the phase transition: e.g., we take $\mu_{ij} = 1$ with probability $(1-p)$ and $\mu_{ij} = -1/2$ with probability p. The transition point shifts but the exponents remain same.

We also conducted detailed simulations for the case where μ_{ij} are quenched variables, i.e., they do not change with time. For the fully connected graph considered here, numerical results for continuous μ_{ij}'s (see Fig. 4) indicate that critical behavior is the same as that of the annealed case mentioned above.

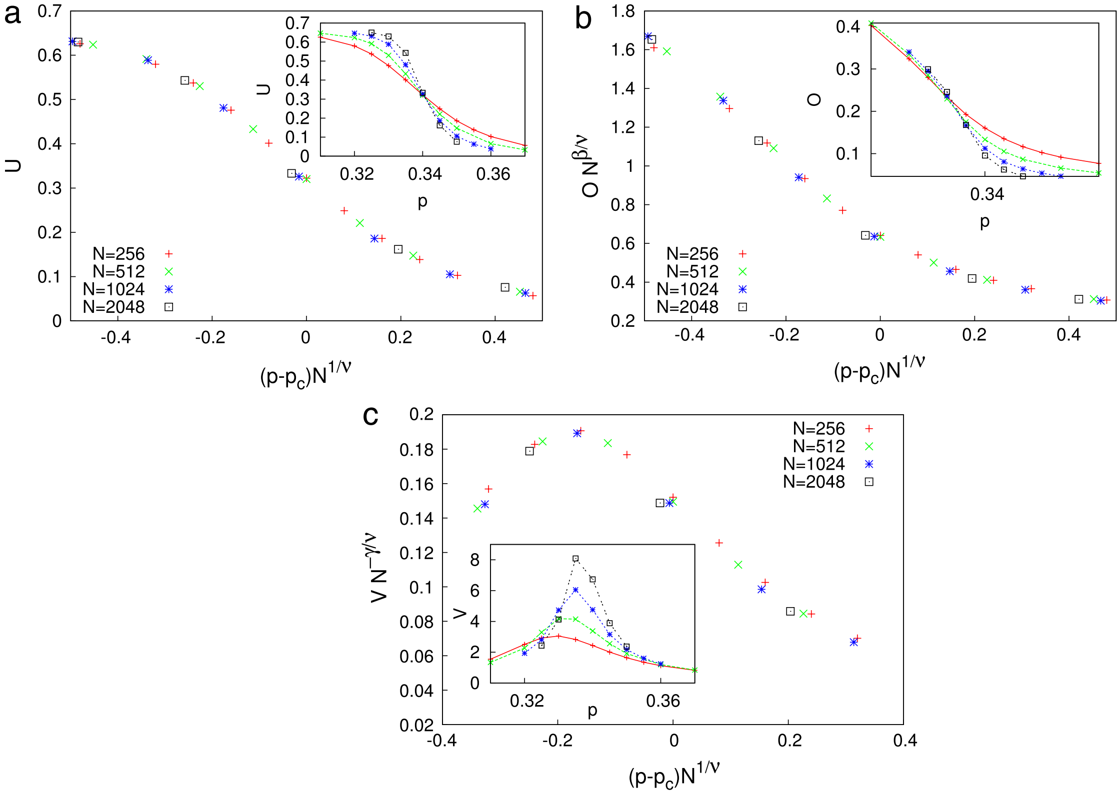

Fig. 4. Data for continuous, quenched μ_{ij} model, showing (a) finite size scaling of the Binder cumulant U for different system sizes N; the critical point is $p_c = 0.34 \pm 0.01$, and the best data collapse is for $\nu = 2.00 \pm 0.01$. Inset: Variation of U with p for different system sizes; (b) finite size scaling of order parameter O for different N; best data collapse is for $\beta = 0.50 \pm 0.01$. Inset: Variation of the order parameter O with p for different system sizes; (c) finite size scaling of V for different N; best data collapse is for $\gamma = 1.00 \pm 0.05$. Inset: Variation of V with p for different N.Number of averages for different system sizes are 3000 for $N = 256$, 1800 for $N = 512$, 1000 for $N = 1024$ and 400 for $N = 2048$.

4. Model with bond dilution

Here we briefly consider the case when all the interactions between the agents are not allowed, i.e., agents are selective in their interactions. In the case of continuous distribution of μ_{ij}, it does not make sense to parametrize the probability of $\mu_{ij} = 0$. It is therefore useful to consider the discrete distribution only for the dilute case by considering $\mu_{ij} = 0$ with probability q along with $\mu_{ij} = -1$ with probability p as before and $\mu_{ij} = 1$ otherwise. Thus, q is naturally the 'bond dilution' parameter.

Considering the balance of the increases and decreases in the order parameter in the steady state, one gets

$$f_1^2 p + f_0 f_1 p + f_0 f_{-1}\left[1-(p+q)\right] = f_{-1}^2 + f_0 f_1\left[1-(p+q)\right] + f_0 f_{-1} p. \tag{10}$$

This gives, either $f_1 = f_{-1}$, i.e., disorder, or in the ordered state

$$f_0 = \frac{p}{1-p-q}. \tag{11}$$

Now, in and out fluxes of f_0 gives

$$f_1^2 p + 2 f_1 f_{-1}\left[1-(p+q)\right] + f_{-1}^2 p = f_0 f_1\left[1-(p+q)\right] + f_0 f_1 p + f_0 f_{-1}\left[1-(p+q)\right] + f_0 f_{-1} p. \tag{12}$$

In the disordered state, the only feasible solution is $f_0 = \frac{1}{3}$. As before, assuming the continuity of f_0 across the transition point, we get the phase boundary equation as

$$p_c = \frac{1}{4}\left(1-q_c\right), \tag{13}$$

where q_c is the critical value of q on the phase boundary. This phase boundary is shown in Fig. 5. Clearly $q = 0$ limit is the unrestricted case studied in the previous section. The fact that the ordered phase extends to $q = 1$ is for $p = 0$ is consistent with the fact that for an infinite dimensional lattice, percolation transition takes place at $q = 1$. The universality of the model along the phase boundary needs to be done. One can expect it to be same as before.

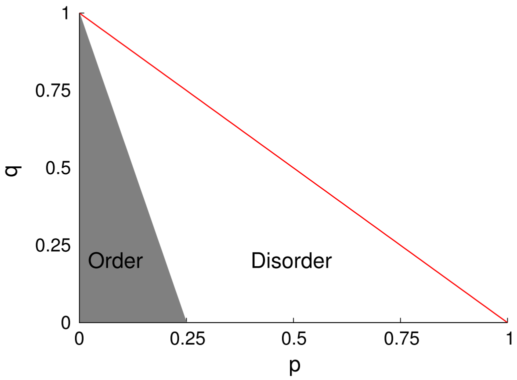

Fig. 5. Phase diagram for the dilute model.

5. Discussions

We present a simplified model for opinion formation in a society of highly connected individuals. With the dynamical rule defined by Eq. (1), an agent modifies his/her opinion under the influence of another. This evolution rule corresponds to cases where opinions are modified following discussions/debates with another individual. The key feature of our model is the inclusion of negative interactions with probability p. We study the steady-state collective behavior of this model. While modeling the opinion formation, the effects of 'self-conviction' and 'external-pressure' have been considered before (see e.g., Ref. [35]).

Here we concentrate upon the study of 'consensus formation' as a dynamical critical phenomenon. Eq. (1) of course does not include all social complexities involved in such interactions, however in this simple form it manifests some intriguing features.

With the introduction of a single parameter p, defined in a simple manner, the proposed model shows the existence of a universal phase transition and also some additional desirable features representing a real society. The parameter p plays a role of the disordering field (similar to temperature in thermally driven phase transitions). Beyond a certain value of the fraction of negative interaction, a phase transition from an 'ordered' state (with most of the individuals having opinions of same sign) to a 'disordered' state (where the opinions can have different signs and add up to zero) occurs (see Eq. (8)).

The highlight of our model (with discrete values of μ_{ij}) lies in the unique selection of an ordered state with discrete opinion values (± 1 and 0) while in the disordered state, all opinion values in $[-1, +1]$ coexist (Fig. 1(c)). The disordered state is one with a lot of disagreement, hence all types of opinions co-exist in the society. But as it starts to get ordered (below p_c), polarization occurs, and marginal opinions cease to exist, resembling the ordering in a multi-party election scenario. This unique selection of the ordered state is in fact independent of the initial conditions which adds to the richness of the model. This of course also happens in the bounded confidence models where opinions, originally varying continuously, ultimately cluster around a few values typically. However, as already mentioned, in these models, the dynamics is designed to achieve this, whereas in our model, it happens naturally without any imposed restrictions.

The phase transition in our model presents a case of a classical continuous phase transition with simple exponent values and showing finite size scaling behavior. To model 'social temperature' which essentially destroys consensus rather than forming it, we have kept negative interactions among the agents. This leads to the desirable feature that even in the 'ordered' phase, opinions of both signs can coexist and that the disordered state is also not a 'neutral state'. In contrast, in the models with only positive interactions [23–25], where each 'scattering' leaves the two agents closer to each others' opinion, finite size behavior was absent and the order of phase transition difficult to identify. Thus, we believe our model is closer to reality and also in terms of critical behavior presents significant modifications.

Another important point to be mentioned is that one could change the distribution of the interactions in a way such that no 'neutral' individual remains in the society (opinion values are ± 1). It is intriguing and to some extent counter intuitive that such processes lead to no consensus in our model (see Eq. (8)). When opinions can take values equal to ± 1 only, a comparison with a Ising spin model is bound to arise. The model with which one should compare is the fully connected Ising model with random bimodal distribution of the interactions ($-J$ with probability p and $+J$ with probability $(1 - p)$). However, in that model there is indeed a phase transition occurring at $p = 1/2$ [33] from a ferromagnetic to a spin glass phase. On the other hand, we get a transition only when the opinions can take more values than just ± 1 (e.g., ± 1 and 0). In our opinion dynamics model, the ordered phase may be regarded as a ferromagnetic phase and the disordered phase as a paramagnetic phase (certainly not a spin glass phase as the opinions do not attain a frozen state had it been so).

When agents are 'selective' in their interactions and some of the interactions are absent or muted, this effectively makes those $\mu_{ij} = 0$. If $\mu_{ij} = 0$ with probability q, then we can conceive a phase diagram with respect to the parameters p and q (see Fig. 5) separating the ordered and disordered phases.

A comparison of our model with the model of Ref. [21] which introduces Galam contrarians (see also Refs. [36–38]) may also be made. In the latter, one has two discrete choices of opinion while the general case of the present model involves continuous opinion values. The mean-field critical behavior, however, is observed in both models with same exponent values. In Ref. [21] the contrarians would take the opinion which is exactly opposite of that of the majority of the group. The evolution of opinions is clearly different in our case, where the original opinion of an agent is also considered while assigning the changed value. This makes even the discrete version of the present model consist of three states $(-1, 0+1)$. Furthermore, we have shown that (see Eq. (9)) if we consider the discrete version to have two states only, then there is no ordered state in the model as soon as any finite fraction of negative interactions is introduced.

Results obtained from opinion dynamics models can be compared to real data to a certain extent. Of course, the microscopic rules governing the dynamics in a model cannot be verified directly but one can justify the model by comparing its macroscopic behavior with real data. Discrete opinion models may be applicable to election scenarios [21]. Continuous opinion models, on the other hand, mimic the case of rating or degree of support for an issue. So in our model, -1 represents extreme unfavorable opinion, $+1$ is extreme favorable while zero means an average rating/indifferent response. Thus the order parameter in the model corresponds to the overall rating and ordered state means there is a clear-cut decision made. A disordered state means the absence of a decision. The situation is comparable to a case of final verdict arrived at by a panel of juries. If there is a lot of disagreement among the juries a decision is hard to achieve—this result is indeed obtained in our model where the disagreement is represented by the negative interaction terms.

Although we have considered continuous opinions in the model, it was shown exactly that using discrete opinions $1, -1, 0$ also leads to the same critical behavior. From the statistical physics viewpoint, the important role in the phase transition is thus played by the parameter p quantifying the fraction of negative interactions, irrespective of the fact that opinions are continuous or discrete. This precisely indicates that p is the relevant parameter in the model and the nature of opinion is irrelevant as far as critical behavior is concerned

We conclude with the remark that the values of the exponents β, γ are very similar to that of the mean field exponents of the Ising model. Interpreting ν as $\nu' d$ where d is the effective dimension in this long ranged model and putting $d = 4$ as is done in small world like networks [39], the value of the effective correlation length exponent $\nu' = 1/2$ also coincides with the mean field value.

For future study, the properties of this model in various lattices and networks and also its dynamical behavior would be interesting [40].

Acknowledgments

The authors would like to specially thank H. Nishimori for very useful correspondence. Inspiring discussions with B. K. Chakrabarti, J. Kertész and M. Marsili are also acknowledged. SB acknowledges discussions with N. Sarkar. PS acknowledges UPE project and DST project for partial support.

References

[1] B.K. Chakrabarti, A. Chakraborti, A. Chatterjee (Eds.), Econophysics and Sociophysics, Wiley-VCH, Berlin, 2006.
[2] D. Stauffer, in: R.A. Meyers (Ed.), Encyclopedia of Complexity and Systems Science, Springer, New York, 2009; Comput. Sci. Eng. 5 (2003) 71.
[3] A.-L. Barabási, Nature 435 (2005) 207.
[4] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, Phys. Rev. Lett. 75 (1995) 1226.
[5] F. Vazquez, P.L. Krapivsky, S. Redner, J. Phys. A 36 (2003) L61.
[6] C. Castellano, S. Fortunato, V. Loreto, Rev. Modern Phys. 81 (2009) 591.
[7] J.E. Cohen, J. Hajnal, C.M. Newman, Stochastic Process. Appl. 22 (1986) 315.
[8] S. Galam, Y. Gefen, Y. Shapir, J. Math. Socio. 9 (1982) 1.
[9] P. Holme, M.E.J. Newman, Phys. Rev. E 74 (2006) 056108.
[10] S. Fortunato, V. Latora, A. Pluchino, A. Rapisarda, Int. J. Mod. Phys. C 16 (2005) 1535.
[11] P.L. Krapivsky, S. Redner, Phys. Rev. Lett. 90 (2003) 238701.
[12] M.N. Kuperman, D.H. Zanette, Eur. Phys. J. B 26 (2002) 387.
[13] T.M. Liggett, Interacting Particle Systems: Contact, Voter and Exclusion Processes, Springer-Verlag, Berlin, 1999.
[14] K. Sznajd-Weron, J. Sznajd, Int. J. Mod. Phys. C 11 (2000) 1157.
[15] S. Galam, Int. J. Mod. Phys. C 19 (2008) 409.
[16] S. Galam, F. Jacobs, Physica A 381 (2007) 366.
[17] G. Deffuant, N. Neau, F. Amblard, G. Weisbuch, Adv. Complex Syst. 3 (2000) 87.
[18] R. Hegselman, U. Krause, J. Art. Soc. Soc. Simul. 5 (2002) 2.
[19] S. Fortunato, Int. J. Mod. Phys. C 16 (2005) 17;
S. Fortunato, Int. J. Mod. Phys. C 15 (2004) 1301.
[20] G. Toscani, Commun. Math. Sci. 4 (2006) 481.
[21] S. Galam, Physica A 333 (2003) 453.
[22] S. Biswas, P. Sen, Phys. Rev. E 80 (2009) 027101;
P. Sen, Phys. Rev. E 81 (2010) 032103;
S. Biswas, P. Sen, J. Phys. A: Math. Theor. 44 (2011) 145003.
[23] M. Lallouache, A.S. Chakrabarti, A. Chakraborti, B.K. Chakrabarti, Phys. Rev. E 82 (2010) 056112.
[24] P. Sen, Phys. Rev. E 83 (2011) 016108.
[25] S. Biswas, A.K. Chandra, A. Chatterjee, B.K. Chakrabarti, J. Phys. Conf. Ser. 297 (2011) 012004.
[26] M. Stone, Ann. Math. Stat. 32 (1961) 1339.
[27] M.H. DeGroot, J. Am. Stat. Asoc. 69 (1974) 118.
[28] S. Chaterjee, E. Seneta, J. Appl. Probab. 14 (1977) 89.
[29] S. Biswas, Phys. Rev. E 84 (2011) 056106.

[30] A. Chakraborti, B.K. Chakrabarti, Eur. Phys. J. B 17 (2000) 167;
A. Chatterjee, B.K. Chakrabarti, S.S. Manna, Physica A 335 (2004) 155;
A. Chatterjee, B.K. Chakrabarti, Eur. Phys. J. B 60 (2007) 135.
[31] A. Chatterjee, Eur. Phys. J. B 67 (2009) 593.
[32] K. Binder, A.P. Young, Rev. Modern Phys. 58 (1986) 801.
[33] H. Nishimori, private communication.
[34] We make use of the special property of the Binder cumulant that it is size invariant at the critical point. e.g., see K. Binder, Z. Phys B: Condens. Matter. 43 (1981) 119.
[35] M. Lewenstein, A. Nowak, B. Latané, Phys. Rev. A 45 (1992) 763.
[36] H.S. Wio, M.S. de la Lama, J.M. Lopez, Physica A 371 (2006) 108.
[37] D. Stauffer, J.S. Martins, Physica A 334 (2004) 558.
[38] S. Galam, Eurphys. Lett. 70 (2005) 705.
[39] H. Hong, B.J. Kim, M.Y. Choi, Phys. Rev. E 66 (2002) 018101;
A. Chatterjee, P. Sen, Phys. Rev. E 74 (2006) 036109.
[40] S. Biswas, A. Chatterjee, P. Sen, in preparation.